Approximation theory in imaging science

Session at the *5th Dolomites Workshop on Constructive Approximation and Applications*

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List of contributions

Schedule

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14:30 – 16:30	G. Vinti F. Dell'Accio V. Bruni D. Poggiali	C. Conti M. Cotronei S. Cuomo F. Marchetti		A. Weinmann M. Storath M. Hockmann	

Time-frequency reassignment methods for non-separable multi-component signals

Vittoria Bruni
University of Roma La Sapienza

Abstract: Modes separation is required in many applications involving non-stationary signals, such as radar and micro doppler systems, seismic signals, audio and human speech signals, animal sounds and biomedical signals. Reassignment method is an effective and automatic non-linear technique providing a sparse time-frequency representation of AM-FM signals; however, its applicability is limited to signals having separable components in the time-frequency plane. In this talk some recent results concerning reassignment methods will be presented with particular reference to approaches oriented to increase the resolution of reassignment images in the case of non-separable components.

Stable recovery of planar images with algebraic boundaries expressed in terms of Bernstein polynomials

Costanza Conti
University of Firenze

Abstract: We present a new method for the stable reconstruction of a class of binary images from sparse measurements. The images we consider are characteristic functions of algebraic domains, that is, domains defined as zero loci of bivariate polynomials, and we assume to know only a finite set of samples for each image. The solution to such a problem can be set up in terms of linear equations associated with a set of image moments. However, the sensitivity of the moments to noise makes the numerical solution highly unstable. To derive a robust image recovery algorithm, we represent algebraic polynomials and the corresponding image moments in terms of bivariate Bernstein polynomials and apply polynomial-reproducing, refinable sampling kernels. This approach is robust to noise, computationally fast, and simple to implement. We illustrate the performance of our reconstruction algorithm from noisy samples through extensive numerical experiments.

This is a joint work with Mariantonia Cotronei (UniRC), Demetrio Labate (University of Huston), Wilfredo Molina (University of Huston)

Image Compression through Multiple Multiresolution Analysis

Mariantonia Cotronei
Universtity of Reggio Calabria

Abstract: We present a strategy for the compression of images based on their multiple multiresolution representation. Such a representation is obtained by processing the data with a tree of filterbanks consisting of filters and decimation matrices that can vary depending on the level. Our algorithm takes advantage of the redundancy of the transformed data by making use of an efficient selection strategy of the portion of coefficients to be kept while still retaining most of the energy of the data. As an extension of standard wavelet and wavelet-like approaches, our method also captures the peculiar anisotropic information of the image while maintaining a low implementation complexity due to its filterbank structure and to the possibility of expressing the employed 2-D filters in an almost separable aspect.

This is a joint work with Doerte Rueweler and Tomas Sauer (Univ. Passau)

An Artificial Intelligence framework for urban zones functional classification

Salvatore Cuomo, Francesco Piccialli
University of Napoli Federico II

Abstract: This talk aims to discuss the problem of Urban Zones Functional Classication (UZFC) via an Artificial Intelligence (AI) approach. More precisely, we construct an AI framework that receives a city's satellite image and produces in the output the functional categories associated with urban areas. This framework is obtained by combining two segmentation procedures: Multiresolution and Selective Search, with three Deep Learning (DL) models: AlexNet, Regularized AlexNet (R-AlexNet) and ResNet-50. The main contributions of our work are: i) evaluating the goodness of our frameworks by comparing them in terms of accuracy; ii) development of an automatic procedure for the ground truth construction in the classication step, which reduces the human effort required by an expert's intervention.

This is a joint work with Fabio Giampaolo, Stefano Izzo, Eduardo Prezioso

References

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2. Kwan Chiman, Ayhan Bulent, Budavari Bence, Lu Yan, Perez Daniel, Li Jiang, Bernabe Sergio and Plaza Antonio, Deep learning for Land Cover Classification using only a few bands, Remote Sensing, 2020.
3. Shorten Connor and Khoshgoftaar Taghi M, A survey on image data augmentation for deep learning, Journal of Big Data, 2019.
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Solving Poisson equation with Dirichlet conditions through multinode Shepard operators

Francesco Dell'Accio
Università degli Studi della Calabria

Abstract: The multinode Shepard operator is a linear combination of local polynomial interpolants with inverse distance weighting basis functions. This operator can be rewritten as a blend of function values with cardinal basis functions, which are a combination of the inverse distance weighting basis functions with multivariate Lagrange fundamental polynomials. The key for simply computing the latter, on a unisolvent set of points, is to use a translation of the canonical polynomial basis and the PA = LU factorization of the associated Vandermonde matrix. In this talk, we discuss a method to numerically solve a Poisson equation with Dirichlet conditions through multinode Shepard interpolants by collocation. This collocation method gives rise to a collocation matrix with many zero entrances and a smaller condition number with respect to the one of the well known Kansa method. Numerical experiments show the accuracy and the performance of the proposed collocation method.

This is a joint work with Filomena Di Tommaso, Otheman Nouisser and Najoua Siar

Convergence analysis in image interpolation in terms of the continuous SSIM

Francesco Marchetti
Università di Padova

Abstract: The Structural Similarity index (SSIM) is widely used in assessing the similarity between two images, which are interpreted as matrices. In this talk, we present and discuss the extension of such index to the continuous framework, that is the continuous SSIM (cSSIM). Then, by relating the cSSIM to the L2-norm, we provide some theoretical results concerning the convergence rates of various image interpolation methods in terms of the cSSIM. Finally, we show some numerical experiments that confirm the theoretical findings.

This is a joint work with Gabriele Santin

Fast algorithms for higher order Mumford-Shah problems based on a Taylor jet formulations

Mathias Hockmann
Osnabruck University

Abstract: Motivated by the application of neural networks in super resolution microscopy, this talk considers super resolution as the mapping of trigonometric moments of a discrete measure on $[0,1)^d$ to its support and weights. We prove that this map satisfies a local Lipschitz property where we give explicit estimates for the Lipschitz constant depending on the dimension d and the sampling effort. Moreover, this local Lipschitz estimate allows to conclude that super resolution with the Wasserstein distance as the metric on the parameter space is even globally Lipschitz continuous. As a byproduct, we present an improved estimate for the smallest singular value of multivariate Vandermonde matrices having pairwise clustering nodes.

Dealing with resampling-induced errors when estimating the activity in clinical multi-imaging

Davide Poggiali
Università di Padova

Abstract:
In modern medical imaging multimodal systems such as (but not restricted to) SPECT/CT, PET/MRI or fMRI/sMRI it is necessary to estimate the activity of the functional image within on or more Volumes of Interest (VOIs) defined over the anatomical reference. Due to a Gibbs effect when oversampling images[1] the popularly-considered rule of thumb is to undersample the VOIs instead. This results in a systemic waste of efforts in high-resolution image segmentation. In this talk we propose to overcome this issue by using the VOIs as a prior information in oversampling. This approach derives from a *Fake Nodes* interpolation scheme[2, 3, 4]. We compare the results of applying vs not-applying such scheme in terms of mean value per VOI, using both simulated images (*in silico*) and using a PET/CT repeated-measures phantom dataset (*in vitro*).

References

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Fast algorithms for higher order Mumford-Shah problems based on a Taylor jet formulations

Martin Storath
Hochschule Würzburg-Schweinfurt

Abstract: Mumford-Shah models are well-established and powerful variational tools for the regularization of noisy data. In the case of images this includes both regularizing the edge set as well as the image values itself. Thus, these models may be used as a basis for a segmentation pipeline or for smoothing the data. We consider higher order Mumford-Shah functionals which penalize the deviation from piecewise polynomials instead of piecewise constant functions as first order Mumford-Shah functionals do. Minimizing Mumford-Shah functionals, which are non-smooth and non-convex functionals, are NP hard problems. Compared with first order Mumford-Shah functionals, numerically solving higher order models is even more challenging, and in contrast to work on more theoretical aspects there are only very few works dealing with the algorithmic side. We propose a new algorithmic framework for second order Mumford-Shah regularization. It is based on a proposed reformulation of higher order Mumford-Shah problems in terms of Taylor jets and a corresponding discretization. Using an ADMM approach, we split the discrete jet-based problem into subproblems which we can solve efficiently, non-iteratively and exactly. We derive numerically stable and fast solvers for these subproblems.

This is joint work with Lukas Kiefer and Andreas Weinmann.

A unifying approach for the study of approximation of nonlinear integral operators

Gianluca Vinti
University of Perugia

Abstract: We study convergence results for a general family of nonlinear integral operator in the case of the pointwise and uniform convergence and in the setting of Orlicz spaces with respect to the modular convergence. Our theory allow us to treat simultaneously, families of sampling type operators as those ones of convolution operators. The first ones have been studied also in connection with concret real world applications.

Iterative Potts Minimization in the Multivariate Case

Andreas Weinmann
Hochschule Darmstadt

Abstract: Signals and images with discontinuities appear frequently in applications in biology, medicine, mechanics, and electrical engineering. The data are typically indirect and noisy measurements of some quantities describing the signal under consideration. A frequent task is to find the segments of the signal / image which corresponds to finding the discontinuities / jumps in the data. For this task, frequently energy minimization methods based on the piecewise constant Mumford-Shah / Potts functional are employed. Due to their non-convexity, minimization of such functionals is challenging. We propose a new iterative minimization strategy for the multivariate Potts functional dealing with indirect, noisy measurements. We present a convergence analysis and illustrate our algorithms with numerical experiments.