Mathematical Imaging and Signal Processing

Program of the working group ”Mathematical Imaging and Signal Processing”
at the Dolomites Research Week on Approximation and Applications 2023 (DRWAA23)

15-19.09.2023, San Vito di Cadore (BL), Italy

Organizers: Wolfgang Erb and Andreas Weinmann

Participants:

- İsmail Aslan (Hacettepe University, TR)
- Ludovico Bruni Bruno (Università degli Studi di Padova, IT)
- Rosanna Campagna (Università di Caserta, IT)
- Cristina Campi (Università di Genova, IT)
- Wolfgang Erb (Università degli Studi di Padova, IT)
- Thomas März (Darmstadt University of Applied Sciences, DE)
- Chiara Razzetta (Università di Genova, IT)
- Andreas Weinmann (Darmstadt University of Applied Sciences, DE)

Schedule of the working group:

Saturday, 16.09.2023

11:15	Rosanna Campagna	*P-spline approximation for instantaneous frequency estimation of non-stationary signals*
12:00	Ludovico Bruni Bruno	*Weights: a spectral analysis*
17:00	İsmail Aslan	*Approximation by Kantorovich type Max-Min Operators with Applications*

Monday, 18.09.2023

11:15	Thomas März	*Harmonic Eigenfunctions: a useful Tool for Reconstruction in MPI*
12:00	Chiara Razzetta	*Delay and Sum beamforming Point Spread Function: local invariance and its consequences*
14:45	Andreas Weinmann	*Algorithms for joint piecewise regression and change point detection*

Tuesday, 19.09.2023

09:30	Wolfgang Erb	*Graph Wedgelets: Adaptive Data Compression on Graphs based on Binary Wedge Partitioning Trees and Geometric Wavelets*

List of contributions

Approximation by Kantorovich type Max-Min Operators with Applications

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Abstract: It is known that nonlinear approximating operators may achieve better approximations results when compared to their linear counterparts [1, 2]. To this aim, Bede et al. construct pseudo-linear operators by changing the algebraic structure of summation and multiplication in the definition of the operators with maximum and minimum operations respectively [1]. Then, Coroianu and Gal examined the Kantorovich type max-product operators in [4], while on the other hand, Duman and Gökçer in [5, 6] obtained a general approximation results for pseudo linear operators of max-min kind. We should remind that these operators are quiet effective in fuzzy logic [3]. On the other hand, although there are a lot of researches about max-product type ones, in the literature, there are only a few papers for max-min kind operators. Initiating from the above studies, in this work, we construct a general form for Kantorovich-type maximum-minimum operators. Then we also study the error of approximation by using Hölder continuous functions and modulus of continuity. Later, as a special case we show that our operator includes Bernstein, Shepard and many other different operators. Moreover, we illustrate our approximations by plotting them in figures. Finally, we apply our approach to obtain increased resolution images.

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Weights: a spectral analysis

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Abstract: Weights are geometrical degrees of freedom for differential forms. They are defined as integral of k-forms over k-submanifolds of a given domain, say a simplex T. This rather intuitive definition yields in turn a large class of interesting problems: how should these supports be chosen? Under whicht hypothesis they are unisolvent? How much interpolation properties are affected by their placement? In the introductory part of this talk, many of these questions will be properly formalised and answered, with a specific focus on unisolvence and symmetry. For these aspects differential geometrical- and algebraic-oriented techniques are involved. As weights yield a generalisation of usual Lagrange interpolator, they are also well suited as degrees of freedom for finite elements. Being FEM a Galerkin method, the error in the approximate solution is unchanged as the finite dimensional space in which the solution is sought is not changed. In contrast, the matrix of the problem depends on the choice of these degrees of freedom, which might then be optimised in order to obtain better spectral properties. We investigate this with the aim of numerical examples, offering an intriguing application to spectral analysis.

P-spline approximation for instantaneous frequency estimation of non-stationary signals

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Abstract: Non-stationary signals consist of the superposition of amplitude and frequency modulated modes. Crucial tasks in their analysis are the separation of each mode and the estimation of the corresponding instantaneous frequency (IF) [1]. Reassignment-like strategies are commonly adopted to address these issues. They consider the time-frequency distribution (TFD) of the signal and mainly attempt to concentrate its energy on the IF curves. Unfortunately, they fail whenever the modes' separability condition is not met. We present a numerical study concerning the approximation of the amplitude modulation function of the time-dependent energy of the signal's TFD, by exploiting a proper sampling of the time variable in the non-separability interval. The proposed study takes advantage of the special form of the time-dependent energy of signal TFD. The latter still is an amplitude modulated signal whose instantaneous frequency depends on modes IFs as well as its amplitude modulation function [2]. P-splines [3] are used to approximate the amplitude modulation function, then used to recover IFs curves in the time-frequency plane. The data regression using a second-order discrete penalty term is improved by constraints that make the P-spline better representing the model stiffness. Moreover, the knots selection for the representation in B-spline basis, can be related to the information about the frequency distribution; this last choice makes the P-spline to capture better the data trend also where information is absent. A preliminary evaluation study is presented, and numerical issues of the whole estimation process are discussed.

References

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Graph Wedgelets: an Adaptive Tool for Signal Processing on Graphs based on Binary Wedge Partitioning Trees

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Abstract: In this talk, we'll introduce graph wedgelets [1] as an adaptive tool for data compression and signal processing on graphs. Graph wedgelets approximate signals on graphs by piecewise constant functions on adaptively generated binary wedge partitionings. In particular, they are discrete variants of continuous wedgelets and binary space partitionings that are frequently used for the compression of 2D images. We prove that continuous results on best m-term approximation with geometric wavelets can be transferred to the discrete graph setting and show that the wedgelet representation of graph signals can be encoded and implemented in a simple way by a binary tree structure. We will also illustrate how this graph-based method can be applied for the compression and segmentation of images.

References

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Harmonic Eigenfunctions: a useful Tool for Reconstruction in MPI

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Abstract: Magnetic Particle Imaging (MPI) is an emerging imaging modality and has become very active field of research. In the multivariate MPI setup images are usually reconstructed using a system matrix which is obtained by a time-consuming measurement procedure. Our approach to the reconstruction problem tkes advantage of a mathematical model which is based on the mathematical description of the MPI signal generation and the properties of the MPI Core Operator. Here, we present a reconstruction algorithm which consits of two major stages: in the first stage, the components of the MPI Core Operator as estimated by using a variational formulation. This variational problem solved by employing an expansion of the solution in harmonic eigenfunctions. In the second stage, the actual image is reconstructed. For this purpose regularized deconvolution is applied to the components of the MPI Core Operator which was robustly estimated in the first stage. We demonstrate the performance of our algorithm with examples.

Delay and Sum beamforming Point Spread Function: local invariance and its consequences

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Abstract: The utilization of medical ultrasound as a non-invasive, real-time imaging system is widespread. It capitalizes on the ability of human tissues to reflect ultrasound signals, allowing for the acquisition of images and quantitative measurements of tissue properties. When considering the various imaging modalities supported by a typical ultrasound apparatus, their characteristics are influenced by numerous factors, both structural and non-structural, of the machine. Specifically, the Point Spread Function (PSF) exhibits spatial variation and relies on the acquisition geometry and probe being used. Moreover, each mode and probe combination involves multiple parameters that impact the quality of the resulting image, such as the transmitted waveform, transmission frequency, and the number of active probe elements. The Delay And Sum (DAS) algorithm is the standard method employed for ultrasound image reconstruction, typically implemented in the hardware of the ultrasound device. Its functionality is dependent on the aforementioned machine parameters. Although this algorithm enables real-time imaging, it poses a limitation when it comes to optimizing parameters to achieve superior reconstructions. To address this, we propose an approximation of the DAS algorithm that reduces the computational cost, thereby making it feasible to optimize acquisition parameters by simulating the image reconstruction process.

Algorithms for joint piecewise regression and change point detection

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Abstract: Finding the change points in time series is a very important and active topic of research which is closely related to fitting a model to the data in between the change points. Numerical algorithms for change point detection frequently perform joint estimation of change points together with the (pieces of the) signal. We report on our results on these topics where we employ variational approaches. Methodologically, we use dynamical programming approaches combined with tailored numerical linear algebra solvers to derive stable and efficient algorithms.