Regularization in Mathematical Imaging

Program of the working group "Regularization in Mathematical Imaging"
at the Dolomites Research Week on Approximation 2019 (DRWA19)

2-6.09.2019, Canazei, Italy

Organizers: Wolfgang Erb, Cristina Campi and Andreas Weinmann

Participants:

- Cristina Campi (Università degli Studi di Padova, I)
- Wolfgang Erb (Università degli Studi di Padova, I)
- Lukas Kiefer (Darmstadt University of Applied Sciences, D)
- Francesco Marchetti (Università degli Studi di Padova, I)
- Thomas März (Darmstadt University of Applied Sciences, D)
- Davide Poggiali (Università degli Studi di Padova, I)
- Andreas Weinmann (Darmstadt University of Applied Sciences, D)

Schedule of the working group:

Monday, 2.09.2019

15:00	Lukas Kiefer	*Direct partitioning of images from indirect data using the Potts model*
15:45	Cristina Campi	*A stopping rule based on risk minimization for iterative methods*
17:00	Andreas Weinmann	*On the variational regularization of inverse problems using nonsmooth functionals*

Thursday, 5.09.2019

11:00	Clemente Cesarano	(special guest)
11:45	Francesco Marchetti	*Variably Scaled Kernels: from image reconstruction to classification tasks*
15:00	Thomas März	*A model-based approach to image reconstruction in magnetic particle imaging*

Friday, 6.09.2019

09:00	Wolfgang Erb	*Compressed sensing in magnetic particle imaging based on sparse sampling along Lissjous curves*

A stopping rule based on risk minimization for iterative methods

Cristina Campi

Dipartimento di Matematica “Tullio Levi-Civita”, Università di Padova,
cristina.campi@unipd.it

In the framework of discrete inverse problems, a vast literature exists on the semi-convergence property, and on how to early stop the iteration, of the Landweber algorithm. Here we investigate the reconstruction error as a function of the number of iterates and we provide a way to write the reconstruction error as a function of the exact data and independently of the unknown object to reconstruct. The outcome is an unbiased estimator of the risk, guaranteeing to select the exact semi-convergent estimate on average. The applications deal with imaging problems, in particular the cases where the signal formation model is a convolution and a Radon transform.

This is a joint work with Federico Benvenuto, Dipartimento di Matematica, Università degli Studi di Genova.

Compressed sensing in magnetic particle imaging based on sparse sampling along Lissjous curves

Wolfgang Erb

Dipartimento di Matematica “Tullio Levi-Civita”, Università di Padova,
erb@math.unipd.it

A common way to acquire system matrices in Magnetic Particle Imaging (MPI) is to measure the MPI system response in a tedious calibration scan at a multitude of positions. In this talk, we propose a compressed sensing approach based on sparse sampling along Lissajous curves to significantly reduce the number of calibration scans. This approach exploits the fact that the rows of the MPI system matrix have a sparse representation in terms of tensor-product Chebyshev polynomials. By using samples along a Lissajous curve we can reformulate this problem into a 1D compressed sensing problem. This 1D problem is then solved by sparsity-promoting optimization algorithms linked to optimal transport.

Direct partitioning of images from indirect data using the Potts model

Lukas Kiefer

Darmstadt University of Applied Sciences
lukas.kiefer@h-da.de

We consider the problem of partitioning images directly from their indirect measurement data without a prior reconstruction. For this task we apply the piecewise constant Mumford-Shah model or Potts model. Since feasible points are piecewise constant, solutions directly yield partitions of the image domain. In this talk, we propose an iterative approach to the NP-hard Potts problem based on surrogate functionals. The algorithmic steps correspond to a gradient descent step and the solution of 1D Potts subproblems that can be solved efficiently by dynamic programming. Furthermore, the 1D problems can be approached in parallel. On the theoretical side, we provide convergence results for our approach.

This is joint work with Martin Storath and Andreas Weinmann

Variably Scaled Kernels: from image reconstruction to classification tasks

Francesco Marchetti

Dipartimento di Salute della Donna e del Bambino, Università di Padova
francesco.marchetti.1@phd.unipd.it

In 2013, Variably Scaled Kernels (VSKs) have been proposed in order to handling the problem of tuning the so-called *shape parameter* in kernel-based interpolation. In this framework, the effectiveness of the reconstruction method is linked to the choice of a proper *shape function*. For example, when dealing with functions presenting jumps, a valid solution is to consider tailored discontinuous shape functions. This intuition led to the formalization of Variably Scaled Discontinuous Kernels (VSDKs), which have been recently applied in the context of Magnetic Particle Imaging (MPI). These adaptive kernels can be considered also in Support Vector Machine (SVM) algorithms. In this view, we present an application in the reconstruction of satellite images and some preliminary results in classification tasks.

A model-based approach to image reconstruction in magnetic particle imaging

Thomas März

Darmstadt University of Applied Sciences
thomas.maerz@h-da.de

Magnetic particle imaging (MPI) is an emerging imaging modality developed by Gleich and Weizenecker in 2005. In this talk We discuss the analysis of a mathematical model of MPI based on the MPI signal encoding. We provide a structured decomposition of the imaging process which enables us to give reconstruction formulae for MPI in 2D and 3D. So far, in the multivariate setup, only time consuming measurement approaches are available,whereas reconstruction formulae are only available in 1D. Our 2D and the 3D reconstruction formulae are significantly different from the 1D situation. In particular, there is no Dirac property in dimensions greater than one in the high resolution limit. In addition, our analysis shows that the reconstruction problem in MPI is severely ill-posed. Finally, we demonstrate a model-based reconstruction algorithm derived from the decomposition of the imaging process.

This is joint work with Andreas Weinmann, Darmstadt University of Applied Sciences.

On the variational regularization of inverse problems using nonsmooth functionals

Andreas Weinmann

Darmstadt University of Applied Sciences
andreas.weinmann@h-da.de

In the context of nonsmooth variational regularization methods prominent examples are total variation models and Mumford-Shah models as well as related higher order methods. In this talk we present parts of our work on this topic including the case of manifold-valued data; in particular, we explain algorithmic approaches and applications.